AD-A068 400 CALIFORNIA INST OF TECH PASADENA DIV OF ENGINEERING--ETC F/G 20/4
UNSTEADY SMALL-GAP GROUND EFFECTS.(U)
MAR 79 E O TUCK N00014-76-C-0157
UNCLASSIFIED E-97B-54 NL

1 OF 1
AD
A068 400

END
DATE
FILMED
6-79
DDC

LEVEL II

(18)

(14) Engineering Science
E-97B-54

AD A068400

(6) UNSTEADY SMALL-GAP GROUND EFFECTS.

by

(10) E. O. Tuck

(9) Technical rept.,

(11) March 1979

(12) 46p.

This research was partially supported by the Office
of Naval Research under Contract N00014-76-C-0157
(15)

Division of Engineering and Applied Science

California Institute of Technology

Pasadena, California 91125

071 585

79 05 03 024

UNSTEADY SMALL-GAP GROUND EFFECTS

by

E. O. Tuck

Applied Mathematics Department
University of Adelaide
SA, Australia

(Work done while on leave at California Institute of Technology and at Massachusetts Institute of Technology)

January, 1978

Flow induced by a body moving near a plane wall is analysed on the assumption that the normal distance from the wall of every point of the body is small compared to the body length. The flow is irrotational except for the vortex sheet representing the wake. The gap-flow problem in the case of unsteady motion is reduced to a non-linear first-order ordinary differential equation in the time variable. Problems solved include airfoil starting flows and their transient wakes, and flat plates falling toward the ground.

1. Introduction

Fluid-dynamic problems involving bodies moving close to walls are of interest in many different contexts, and there is a considerable literature dealing with such problems. The present paper concerns itself with phenomena which can be treated on an inviscid-fluid basis, and hence has little connection with the important branch of that literature dealing with low-Reynolds-number wall effects. (cont on p 3)

The classical inviscid problem is of course the aerodynamics of wings near the ground, and indeed the present work has its most obvious application in that area. The regime of interest in the present paper is, however, that of very small clearances to the ground. The early work on aerodynamic ground effect (e.g. as surveyed by Pistolesi, 1937), views all effects of the ground as small perturbations to the infinite-fluid flow about the wing. The formal requirement for this to be valid is that the clearance between body and ground be large compared to all length scales of the body.

An intermediate regime in which the flow is qualitatively similar to that in an infinite fluid, but where ground effects are "0(1)" perturbations, is when the clearance is comparable to some body dimension. For example, (e.g. Bagley, 1961, Tuck and Newman, 1974) the problem of a thin airfoil in steady ground effect at clearances comparable to the chord (thus large compared to the thickness and/or camber), leads to a singular integral equation for the bound vorticity, whose kernel reduces to the classical lifting-surface kernel for an unbounded fluid, as the clearance/chord ratio

tends to infinity. On the other hand, if one lets the clearance/chord ratio tend to zero in such a theory, the integral equation "collapses", and one obtains an almost-trivial explicit result for the loading, in terms of the local distance from the wall.

(cont'd p 2)

The small-gap regime is defined, formally, as that in which the clearance is small compared to the horizontal length scale. (cont on p 1473A) For non-thin bodies (e. g. automobiles), a general approach to this class of problem is outlined by Tuck (1975); however, as with any bluff-body flow, little progress can be made without introducing empirical assumptions regarding the wake. We assume here that, in addition, the body is thin, i.e. that not only its lower surface, but also its upper surface, is close to the wall.

Some literature does exist on the small-gap problem. For example, Strand, Royce and Fujita (1962) noted the "hydraulic" or channel-flow character of the tightly-constrained flow between the body and the wall, and Widnall and Barrows (1970) provided a complete asymptotic solution for the steady-flow case, assuming in addition that the thickness and camber (and "angle of attack times chord") are small compared to the clearance. The present paper can be considered as an extension of the work of Widnall and Barrows to include "non-linear" (thickness etc. comparable to clearance) and unsteady effects. Unsteadiness due to ground irregularity was included in the linear theory by Barrows and Widnall (1970).

Although the potential applications are to very practical problems, such as tracked ground vehicles (Barrows, 1971), and ship manoeuvring in shallow water near banks (Norrbin, 1974; Tuck, 1978), in the present paper we give only simplified illustrations, involving two-dimensional flow, and specialized geometry and motions. The small-clearance assumption implies that a two-dimensional flow becomes one-dimensional in the gap, and is thus describable by the one-dimensional continuity equation. For a given body under-surface, this equation is a second-order ordinary differential equation for the velocity potential, as a function of the horizontal coordinate x along the wall, which can be solved explicitly. Two boundary conditions are needed, and these must come from matching with the outer flow passing over the top of the body. In the Appendix, we show that the proper conditions are continuity of velocity potential at the leading edge, and (for bodies without stern appendages) of pressure at the trailing edge.

When these conditions are applied in unsteady flow, the result is a non-linear ordinary differential equation of the first order with respect to time, to determine the net flux exiting from the gap at the stern. In the special case of steady flow, the appropriate solution of this equation is vanishing flux, in a frame of reference fixed in the fluid at infinity. That is, in a frame of reference fixed in the body, the trailing-edge velocity is equal to the free-stream velocity. The present theory then reduces to a non-linear equivalent of that of Widnall and Barrows (1970). Some non-linear consequences for the steady lift force are discussed in §4.

In §5 we analyse the transient establishment of the steady flow of §4, starting from rest. The differential equation for the gap flux at the stern can be solved explicitly, for the case of an impulsive start at constant speed. In this simple case, we can also solve for the velocity and thickness of the vortex wake left behind the body, and, for example, predict a shock-like rolling up of the wake behind an airfoil at a negative angle of attack.

The problem solved in §6 is free fall under its own weight of a flat plate toward a plane wall, combined with a horizontal motion at constant speed U. For $U = 0$, a symmetric solution was presented by Yih (1974), in which the plate never actually hits the wall, but ultimately approaches it with an exponential decay of height with time. We find a similar result, but only for U above a certain critical value. For lower values of U, the approach to the wall is still gentle, but there is impact at a finite time, with a cubic decay of height with time near impact. It should be noted, however, that we retain the asymmetry between leading and trailing edges even when U is small, and Yih's symmetric solution is likely to be more realistic for small U. We also assume that the angle of attack remains zero for all time, and (at least when $U \neq 0$) this makes the application to problems such as stacking of glass plates (Yih, 1974) or the "sliding of sheets of paper" illustration in G. I. Taylor's movie (1967) not yet complete. It would appear, however, that there is no need to invoke viscous effects to explain a number of air-lubrication phenomena.

Indeed, in many respects the present paper merely opens the door to further investigation of interesting small-gap phenomena. The potential applications to ship manoeuvring are particularly important. Although we have phrased the theory here in terms of wall effects, there is also no difficulty in extending it to the more general problem of two-body interactions, e.g. ships in passing manoeuvres, or biplane wings.

Extensions to three-dimensional flow can take a number of forms. Perhaps of greatest practical interest is the case where a ship is moving in shallow water with a very small (compared to its beam) clearance to the bottom, and a somewhat larger clearance (e.g. comparable to its beam) to a side wall or another ship. In that case, the one-dimensional assumption remains valid between ship and wall, but there is possible leakage of fluid under the ship, i.e. out of the "channel". This geometry is similar to that considered by Barrows (1971) for a ram wing in a "trough".

2. Problem Formulation

We assume two-dimensional irrotational flow of an incompressible fluid, generated by movement of a thin airfoil-like body as sketched in Figure 1, with upper surface $y = f_U$, and lower surface $y = f_L$, i.e. occupying the region

$$f_L(x,t) < y < f_U(x,t) , \qquad x_B(t) < x < x_S(t) . \tag{2.1}$$

The leading edge or bow is at $x = x_B$, and the trailing edge or stern at $x = x_S$. The body has length

$$2\ell = x_B - x_S , \tag{2.2}$$

which can in the most general case depend on time, but is normally considered to be constant.

The body is everywhere close to a plane boundary at $y = 0$, i.e.

$$f_L , f_U = O(\epsilon \ell) \tag{2.3}$$

where ϵ is a small parameter. This also requires the body to be thin, as in the thin-airfoil theory. However, we do not assume that its thickness $f_U - f_L$ is small compared to the wall separation, e.g. to the minimum value of f_L.

Our task is to solve Laplace's equation

$$\phi_{xx} + \phi_{yy} = 0 \tag{2.4}$$

for the velocity potential $\phi(x, y, t)$, subject to suitable boundary conditions. At infinity we have a state of rest, i. e.

$$\phi, \ \underset{\sim}{\nabla}\phi \to 0 \qquad \text{as } x, y \to \infty. \tag{2.5}$$

The wall $y = 0$ is impermeable, i. e.

$$\phi_y(x, 0, t) = 0 . \tag{2.6}$$

The boundary conditions on the moving body surface are

$$\phi_y = \dot{f} + \phi' f' , \tag{2.7}$$

where we use a dot for $\frac{\partial}{\partial t}$ and a prime for $\frac{\partial}{\partial x}$. Equation (2. 7) applies both with $f = f_L$ and $f = f_U$.

Finally we must use an appropriate wake and Kutta condition. The trailing edge $x = x_S$ is assumed to shed vortices which remain behind the body in a vortex sheet, with equation

$$y = f_W(x, t) , \tag{2.8}$$

the function $f_W(x, t)$ being unknown. The kinematic boundary condition across this surface is given again by (2. 7), with $f = f_W$. The dynamic condition is continuity of pressure across the sheet. Thus if $p(x, y, t)$ is the excess of pressure over the value at infinity, then from

Bernoulli's equation

$$p = -\rho(\dot{\phi} + \tfrac{1}{2}\phi_x^2 + \tfrac{1}{2}\phi_y^2) \tag{2.9}$$

everywhere in the fluid. The dynamic boundary condition on the wake is

$$p(x, f_W + 0, t) = p(x, f_W - 0, t) . \tag{2.10}$$

Equation (2.10) holds in the wake for $x > x_S(t)$ and also holds at the trailing edge $x = x_S(t)$, where it defines the Kutta condition, and ultimately determines the circulation about the body.

3. One-Dimensional Theory

We now assume that in the limit as $\epsilon \to 0$, $\phi = O(\epsilon)$ except in the near-wall gap region G defined by

$$G: \quad 0 < y < f(x,t) , \qquad x > x_B(t) , \tag{3.1}$$

where

$$f(x,t) = \begin{cases} f_L(x,t) , & x_B(t) < x < x_S(t) \\ f_W(x,t) , & x > x_S(t) . \end{cases} \tag{3.2}$$

In the region G between the body (or its wake) and the wall, there is a flow of a magnitude which does not tend to zero as $\epsilon \to 0$. If we are interested only in leading-order estimates of forces, etc., we can therefore concentrate attention on the flow in region G, assuming in effect that $\phi \equiv 0$ when we are outside G. A justification for this assumption is provided in the Appendix by matching techniques (see also Widnall and Barrows, 1970).

In fact, the flow in G is classical one-dimensional or channel flow, in which the fluid moves predominantly in the x direction, parallel to the wall. For example it is consistent with this approximation to use the Taylor expansion

$$\phi = \phi(x,0,t) - \tfrac{1}{2} y^2 \phi''(x,0,t) + \dots . \tag{3.3}$$

which guarantees satisfaction of (2. 4) and (2. 6), and satisfies (2. 7) if

$$\dot{f} + (f\phi')' = 0 . \tag{3. 4}$$

Alternatively, one may recognize (3. 4) as the ordinary one-dimensional continuity equation, expressing conservation of mass in a flow with dominant velocity $\phi'(x, 0, t)$, in a channel of width f which varies both in space and time. From now on, we write $\phi(x, t)$ for $\phi(x, 0, t)$.

One integration of (3. 4) gives

$$f(x, t)\phi'(x, t) = q(t) + \int_x^{x_S(t)} \dot{f}(\xi, t)d\xi , \tag{3. 5}$$

where

$$q(t) = f(x_S(t), t)\phi'(x_S(t), t) \tag{3. 6}$$

is the (so-far unknown) net flux through the gap at the stern. Our primary task is to determine this quantity $q(t)$. A further integration gives the potential ϕ itself. In performing this integration step, we match with the exterior flow $\phi \equiv 0$ outside G, by applying the boundary condition

$$\phi(x_B(t), t) = 0 , \tag{3. 7}$$

at the leading edge $x = x_B(t)$. This is justified formally in the Appendix. Thus

$$\phi(x,t) = q(t)A(x,t) + B(x,t) \ , \tag{3.8}$$

where

$$A' = \frac{1}{f} \ ,$$

i.e.

$$A(x,t) = \int_{x_B(t)}^{x} \frac{d\xi}{f(\xi,t)} \ , \tag{3.9}$$

and

$$B' = \frac{1}{f} \int_{x}^{x_S} \dot{f}\, d\xi \ ,$$

i.e.

$$B(x,t) = \int_{x_B(t)}^{x} \frac{d\xi}{f(\xi,t)} \int_{\xi}^{x_S(t)} \dot{f}(\xi^*,t)\, d\xi^* \ . \tag{3.10}$$

For a given body lower surface $f = f_L$, (3.8) determines the flow in that part of G beneath the body, providing we can find $q(t)$. In order to find $q(t)$, we must make use of the Kutta condition (2.10) at the trailing edge $x = x_S(t)$. Now in G , we can simplify the Bernoulli equation (2.9) since $\phi_y = O(\epsilon)$, and hence the pressure in G is given by

$$p(x,t) = -\rho\left[\dot{\phi}(x,t) + \tfrac{1}{2}(\phi'(x,t))^2\right] + O(\epsilon^2) . \tag{3.11}$$

Since $\phi \equiv 0$ outside G , and hence $p \equiv 0$ (actually $p = O(\epsilon)$) , continuity of p across the wake simply requires p to vanish there, i.e.

$$\dot{\phi}(x,t) + \tfrac{1}{2}(\phi'(x,t))^2 = 0 , \quad x > x_S(t) . \tag{3.12}$$

Equation (3.12) is a first-order non-linear partial differential equation to determine the flow in the wake region, equivalent to the homogeneous Euler equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \tag{3.13}$$

for the velocity $u = \phi'$. Once (3.12) or (3.13) is solved, we have another first-order equation (3.4) to solve for the unknown wake boundary $y = f_W(x,t)$. Such a computation is presented in §5. However, it is not necessary to go this far, if we are primarily interested only in the flow beneath the body.

So long as the gap and wake regions are essentially continuous with each other (i. e. not interrupted by an edge region where ϕ_y is significant), equation (3. 12) can also be applied "at" $x = x_S(t)$, and is the required Kutta condition to determine the trailing-edge flux $q(t)$. In the Appendix we show that, if the under-surface at the trailing-edge is "shaped", or if there is an appendage such as a rudder at a finite angle of attack, (3. 12) is replaced by

$$\dot{\phi} + \tfrac{1}{2}\phi'^2 = \frac{\sigma^2 - 1}{2\sigma^2}(\dot{x}_s - \phi')^2 , \qquad (3.14)$$

where σ is the contraction coefficient of the "nozzle" formed between the trailing edge and the ground plane. If $\sigma = 1$, as we assume henceforth, (3.14) reduces to (3. 12) at $x = x_S$.

On substitution of the representation (3. 8) for the velocity potential ϕ into the Kutta condition (3. 12) at $x = x_S(t)$, we obtain the non-linear first-order ordinary differential equation

$$A(x_S, t)\dot{q} + \dot{A}(x_S, t)q + \dot{B}(x_S, t) + \frac{q^2}{2f^2(x_S, t)} = 0 . \qquad (3.15)$$

Note that the coefficients $\dot{A}$, $\dot{B}$ are obtained by differentiating the corresponding expressions (3. 9), (3. 10) with respect to time, and <u>then</u> setting $x = x_S(t)$; in particular, $\dot{A}(x_S, t) \neq \frac{d}{dt}A(x_S, t)$. The differential equation (3. 15) must be solved in any special case, subject to a suitable initial condition on the flux $q(t)$ at $t = 0$.

The above applies for an arbitrarily-deforming body surface. We are most interested in the case of rigid bodies with 3 degress of freedom, measured by the co-ordinates $(x_o(t), y_o(t))$ of the mid-point of the body, and the (bow-up) pitching angle $\theta(t)$ about that point. If (X, Y) are co-ordinates fixed in the body, aligned with the (x, y) axes at $\theta = 0$, and if

$$Y = f_o(X), \qquad |X| < \ell, \tag{3.16}$$

is the equation of the lower surface of the body in these co-ordinates, then when motion occurs, the lower surface is given by $y = f(x,t)$, where

$$f(x,t) = y_o(t) - \theta(t)(x - x_o(t)) + f_o(x - x_o(t)). \tag{3.17}$$

On substitution into (3.9), (3.10), we have

$$A(x,t) = \int_{-\ell}^{x-x_o} \frac{d\xi}{y_o - \theta\xi + f_o(\xi)}, \tag{3.18}$$

and

$$B(x,t) = \int_{-\ell}^{x-x_o} \frac{d\xi}{y_o - \theta\xi + f_o(\xi)} \Big[(\dot{y}_o + \theta\dot{x}_o)(\ell - \xi) - \tfrac{1}{2}\dot{\theta}\ell^2 - \dot{x}_o f_o(\ell) + \tfrac{1}{2}\dot{\theta}\xi^2 + \dot{x}_o f_o(\xi)\Big]. \tag{3.19}$$

4. Steady Flow

An important special case is when $x_o = -Ut$, $y_o = \theta = 0$. That is, the rigid body with lower surface $y = f_o(x + Ut)$ is moving with constant velocity U in the negative x-direction. Although it is somewhat easier to solve this problem directly in the moving (X, Y) co-ordinate system, in which the body appears fixed in a uniform stream of magnitude U in the $+x$ direction, we provide the solution here by specialization of the general unsteady results.

We observe that

$$A = \int_{-\ell}^{x+Ut} \frac{dX}{f_o(X)} , \tag{4.1}$$

and

$$B = Uf_o(\ell)A - (x + Ut + \ell)U , \tag{4.2}$$

and find that $\dot{B} = 0$ at the stern. Hence the differential equation (3.15) possesses the solution $q = 0$, which simply means that free-stream conditions prevail at the stern. Equation (3.8) then shows that (4.2) is in fact an expression for the velocity potential $\phi(x,t)$ everywhere in the gap, from which we can compute (using (3.11)) the pressure in the gap,

$$p = \frac{1}{2}\rho U^2 \left[1 - \frac{f_o^2(\ell)}{f_o^2(x + Ut)} \right] . \tag{4.3}$$

Since we have assumed that $p = 0(\epsilon)$ on the upper surface of the body, the net upward force per unit span is

$$F_L = \rho U^2 \ell (1 - \overline{\lambda^{-2}}) + 0(\epsilon) , \tag{4.4}$$

where

$$\lambda(x) = f_o(x)/f_o(\ell) \tag{4.5}$$

is the gap thickness, scaled with respect to that at the trailing edge, and a bar denotes a mean value over the length of the body, i. e.

$$\overline{\lambda^{-2}} = \frac{1}{2\ell} \int_{-\ell}^{\ell} \frac{dx}{(\lambda(x))^2} . \tag{4.6}$$

The result (4. 4) is equivalent to one obtained by Strand, Royce and Fujita (1962).

It is instructive to compare the above lift force with that which would be obtained by use of the Kutta-Joukowski theorem, which applies only in unbounded media. Thus (since ϕ is continuous elsewhere), the net circulation Γ around the body is generated entirely by the jump in the velocity potential ϕ, from its value $\phi(x_S, t)$ immediately below the trailing edge, to its zero value immediately above it. Hence

$$\begin{aligned}\Gamma &= \phi(x_S, t)\\ &= -2U\ell + Uf_o(\ell)\int_{-\ell}^{\ell}\frac{dx}{f_o(x)}\\ &= -2U\ell(1-\overline{\lambda^{-1}})\ .\end{aligned} \tag{4.7}$$

Thus the Kutta-Joukowski lift is

$$\begin{aligned}F_K &= -\rho U\Gamma\\ &= 2\rho U^2\ell(1-\overline{\lambda^{-1}})\ .\end{aligned} \tag{4.8}$$

In cases where the body's thickness or camber is small compared to the wall separation, the Kutta-Joukowski theorem becomes asymptotically valid, and $F_K \rightarrow F_L$, since in that limit ($f_o(x) \approx f_o(\ell)$)

$$\begin{aligned}1-\overline{\lambda^{-2}} &\rightarrow 2(1-\overline{\lambda^{-1}})\\ &\rightarrow 2(\overline{\lambda}-1)\ .\end{aligned} \tag{4.9}$$

That is,

$$F_L \rightarrow 2\rho U^2\ell(\overline{\lambda}-1)\ , \tag{4.10}$$

which depends only on the net area $2\ell(\overline{\lambda}-1)$ lying between the lower surface of the body and a horizontal line through its trailing edge. For example, if the body is a flat plate at an angle of attack α, then

$$F_L = \tfrac{1}{2}\rho U^2 . 2\ell . a. \left(\frac{2\ell}{f_o(\ell)} \right) , \qquad (4.11)$$

and is inversely proportional to the clearance/chord ratio. The result (4. 11) was found by Widnall and Barrows (1970).

5. Impulsively-Started Body at Zero Angle of Attack

We assume again that $\theta = 0$, $y_o = 0$, and hence the gap thickness is

$$f(x,t) = f_o(x - x_o(t)) . \qquad (5.1)$$

The differential equation (3. 15) becomes

$$A_o\dot{q} - \frac{\dot{x}_o}{f_o} q + \ddot{x}_o(2\ell - f_o A_o) + \frac{q^2}{2f_o^2} = 0 , \qquad (5.2)$$

where

$$A_o = \int_{-\ell}^{\ell} \frac{d\xi}{f_o(\xi)} = 2\ell\,\overline{\lambda^{-1}}/f_o(\ell) , \qquad (5.3)$$

and

$$f_o = f_o(\ell) . \qquad (5.4)$$

For a general pattern $x_o(t)$ of horizontal motion, we must solve (5. 2) numerically. However, for the special case

$$\ddot{x}_o = - U\delta(t) , \qquad (5.5)$$

in which there is a sudden commencement of steady motion toward the left at speed U , we have to solve

$$A_o \dot{q} + \frac{U}{f_o} q + \frac{q^2}{2f_o^2} = 0 , \tag{5.6}$$

subject to the initial condition

$$q(o) = U\left(\frac{2\ell}{A_o} - f_o\right) , \tag{5.7}$$

which follows by integrating (5.2) from $t = 0-$ to $t = 0+$. The solution is given by

$$\frac{2Uf_o}{q} = \mu e^{t/t_o} - 1 , \tag{5.8}$$

where

$$\mu = (1 + \overline{\lambda^{-1}}) / (1 - \overline{\lambda^{-1}}) , \tag{5.9}$$

and

$$\begin{aligned} t_o &= f_o A_o / U \\ &= 2\ell\, \overline{\lambda^{-1}} / U \\ &= \frac{f_o(\ell)}{U} \int_{-\ell}^{\ell} \frac{d\xi}{f_o(\xi)} . \end{aligned} \tag{5.10}$$

As $t \to \infty$, $q \to 0$, and the steady flow of §4 is recovered. The time scale for development of the steady flow is measured by t_o,

e.g. if $\overline{\lambda^{-1}} = \frac{1}{2}$ $(\mu = 3)$, then the flux has fallen to 28% of its starting value when $t = t_o$. It should be noted that during any time interval t_o, the body moves a fraction $\overline{\lambda^{-1}}$ of its length. However, if it happens that $\overline{\lambda^{-1}} = 1$ $(\mu = \infty)$, the steady flow is established immediately, since $q(o) = o$ implies $q = o$ for all $t > o$. This applies to some non-trivial flows, as well as to the trivial zero-disturbance case $\lambda(x) \equiv 1$, of a flat bottom without angle of attack.

In the present case it is straightforward to solve (3. 13), (3. 4) for the wake velocity $\phi' = u(x,t)$ and wake width $f = f_W(x,t)$, the problem being essentially identical to that for a compressible gas at constant pressure, as described for example by Ames (1965, p. 51), with f_W playing a role analogous to the gas density ρ. Thus the general solution of (3. 13) is given by the transcendental equation

$$x = ut + G(u) , \tag{5. 11}$$

for some function $G(u)$. Applying the boundary condition

$$u = \frac{q(t)}{f_o} \quad \text{at} \quad x = -Ut , \tag{5. 12}$$

where $q(t)$ is given by (5. 8), we find

$$G(u) = -t_o (U + u) \log \left| \frac{2U + u}{\mu u} \right| . \tag{5. 13}$$

The corresponding general solution of the linear equation (3. 4) for f is

$$f_W(x, t) = \frac{H(u)}{t + G'(u)} \quad . \tag{5.14}$$

for some function $H(u)$, determined from the boundary condition

$$f_W = f_o \text{ at } x = -Ut , \tag{5.15}$$

to be

$$H(u) = f_o \left[G'(u) - \frac{G(u)}{U+u} \right]$$

$$= 2f_o \, Ut_o \frac{U+u}{u(2U+u)} \quad . \tag{5.16}$$

In Figure 2, we plot u and f_W against x, for various values of t, with $\mu = 3$, i.e. $\overline{\lambda^{-1}} = \frac{1}{2}$. This is a case with effectively-positive angle of attack, and hence acceleration of the fluid in the gap. There is positive stern flux for all time, and hence the vortices shed at the trailing edge move to the right, and the wake continually lengthens and thins.

A very different picture emerges when μ is negative, or $\overline{\lambda^{-1}} > 1$, i.e. when there is an effective negative angle of attack. Now there is deceleration in the gap, and negative flux at the stern. Hence the shed vortices move to the left, and the wake rolls up on itself. This is displayed in the mathematics, by the possibility of a singularity in (5.14) if $t = -G'$, which can happen only if $\mu < 0$. The phenomenon is similiar to shock waves in gas dynamics.

For example, in Figure 3 we present results for $\mu = -3$, i.e. $\overline{\lambda^{-1}} = 2$. The shock first appears at $t/t_o = 4/3$; subsequently $f_W \rightarrow \infty$ at values of x to the left of a point moving with the starting value of u (which is also the wake-edge value for all time, in the absence of shocks). Further discussion of this interesting phenomenon is clearly beyond the scope of the present paper.

6. Falling Flat Plate in Steady Horizontal Motion

We now assume $f_o(x) = 0$, $\theta = 0$, and $x_o = -Ut$, but allow an arbitrary vertical movement $y = y_o(t)$. The velocity potential (3. 8) becomes

$$\phi = u(x + Ut + \ell) + \frac{1}{2}\beta(x + Ut + \ell)(x + Ut - 3\ell) \tag{6. 1}$$

where

$$u = q/y_o = \phi'(-Ut + \ell, t) \tag{6. 2}$$

is the gap velocity at the stern, and

$$\beta = -\dot{y}_o/y_o . \tag{6. 3}$$

The differential equation (3. 15) becomes

$$2\ell\dot{u} - 2\ell^2\dot{\beta} + Uu + \frac{1}{2}u^2 = 0 . \tag{6. 4}$$

For any <u>given</u> time history of vertical movement $y_o(t)$, we must evaluate the quantity $\beta(t)$, and then solve the differential equation (6. 4) for $u(t)$.

The pressure on the plate's under surface is given by

$$-\frac{p}{\rho} = \dot{u}(x + Ut + \ell) + \frac{1}{2}\dot{\beta}(x + Ut + \ell)(x + Ut - 3\ell) + U[u + \beta(x + Ut - \ell)]$$
$$+ \frac{1}{2}[u + \beta(x + Ut - \ell)]^2 , \tag{6. 5}$$

and vanishes as required at $x + Ut = \ell$, if (6.4) is satisfied. The net upward force is

$$F_L = -\rho\ell[2\ell\dot{u} - 3\ell^2\dot{\beta} + 2U(u - \beta\ell) + (u - \beta\ell)^2 + \frac{1}{3}\ell^2(\dot{\beta}+\beta^2)] \quad (6.6)$$

and the (bow-down) moment is

$$M = -\frac{2}{3}\rho\ell^3[\dot{u} - \ell\dot{\beta} + (U + u)\beta - \ell\beta^2] \; . \quad (6.7)$$

For example, consider a freely-falling plate of mass m per unit span, constrained against rotation. Then $y_o(t)$ itself is determined by solving the equation of motion

$$F_L = m(\ddot{y}_o + g) \; , \quad (6.8)$$

i.e.

$$2\ell\dot{u} - \frac{8}{3}\ell^2\dot{\beta} + 2Uu + u^2 - 2(U + u)\beta\ell + \frac{4}{3}\ell^2\beta^2$$

$$= -\frac{m}{\rho\ell}(\ddot{y}_o + g) \; . \quad (6.9)$$

Equations (6.3), (6.4) and (6.9) are a set of 3 coupled non-linear first-order differential equations to solve for u, β and y_o.

If we define $t = t_o$ as the instant at which the plate hits the wall, so that motion takes plate for $t < t_o$, then so long as t_o remains finite, the final stage of the fall appears to be described by the estimates

$$u = -\frac{2\ell}{t-t_o} - U - \left[\frac{mg}{4\rho\ell^2} + \frac{U^2}{6\ell}\right](t-t_o) + 0(t-t_o)^2 , \tag{6.10}$$

$$\beta = \frac{-3}{t-t_o} - \frac{U^2(t-t_o)}{4\ell} + 0(t-t_o)^2 , \tag{6.11}$$

and

$$y_o = -c\,(t-t_o)^3\,[1 + \frac{U^2(t-t_o)^2}{8\ell^2} + 0(t-t_o)^3] \quad , \tag{6.12}$$

for some constant c. This constrasts with the exponential approach to the wall found by Yih (1974) at $U = 0$, by making the empirical assumption that $p = 0$ at both edges of the plate. This means that vortex sheets must spring from both edges, there being of course no distinction between "leading " and "trailing" edges if $U = 0$. Although the result is different, the conclusion that the approach to the wall is relatively "gentle" still holds! However, the requirement of no rotation is rather restrictive, since the limiting bow-down moment is

$$M = \frac{8}{3}\,\frac{\rho\ell^4}{(t-t_o)^2} + 0(1) . \tag{6.13}$$

In fact it is possible to have an exponential approach to the wall, for sufficiently-large horizontal speed U. Thus, the system (6. 3), (6. 4), (6. 9) allows a solution with $u \to 0$, $\beta \to \beta_o$, and

$$y \to c\,e^{-\beta_o t} \to 0 , \quad t \to \infty , \tag{6.14}$$

where β_o is a positive constant determined from the quadratic equation

$$-2\,U\ell\,\beta_o + \frac{4}{3}\ell^2\beta_o^2 = -\frac{mg}{\rho\ell} \qquad (6.15)$$

$$\text{i.e.}\quad \beta_o = \frac{3U}{4\ell} \pm \sqrt{\frac{9U^2}{16\ell^2} - \frac{3mg}{4\rho\ell^3}}\ . \qquad (6.16)$$

This solution possesses a finite bow-up limiting moment M.

The exponentially-decaying solution can exist only if the square root in (6.16) is real, i.e. if

$$U > \sqrt{\frac{4mg}{3\rho\ell}} = U_o\ . \qquad (6.17)$$

The value of β_o at this limiting speed U_o is one half of the value found by Yih at $U = 0$, and as $U \to \infty$, the dominant (smaller β_o) decaying term has $\beta_o \to 0$. The present theory must break down for sufficiently small U, and Yih's theory appears a suitable extrapolation to $U = 0$, with the advantage of flow symmetry about the centre of the plate.

The above system was solved numerically for motion starting from rest at $y_o = h$. We use a non-dimensional version of the equations, namely

$$\dot{y}_o = -\beta y_o \qquad , \ y(o) = 1 \ ,$$

$$\dot{u} = \dot{\beta} - \frac{1}{2} Uu - \frac{1}{4} u^2 \qquad , \ u(o) = 0 \ ,$$

$$\dot{\beta} = \frac{-\frac{1}{2} Uu + U\beta - \frac{1}{4}u^2 + u\beta - \frac{2}{3}\beta^2 - \frac{1}{2} my_o \beta^2 - m}{-\frac{1}{3} - \frac{1}{2} my_o} \ , \ \beta(o) = 0, \qquad (6.18)$$

where the scale for time is the time taken to fall a distance h in a vacuum, i.e.

$$T = \sqrt{\frac{2h}{g}} \ , \qquad (6.19)$$

and y_o is scaled with h, u and U with ℓ/T, β with $1/T$, and m with $\rho\ell^3/h$.

It appears from these computations that whenever the condition (6.17) is satisfied (i.e. high speed U), the exponential-decay solution holds, with the smaller of the two values of β_o in (6.16). On the other hand, if (6.17) is not satisfied (low speed U), the body does indeed hit the ground at a finite time t_o, and $t_o \to \infty$ as $U \to U_o$.

Figure 4 shows a typical set of numerical results at (scaled) $m = 1$, for various values of (scaled) U. The critical scaled speed is $U_o = \sqrt{8/3} = 1.64$ in this case. For example, we find at $U = 1$ that the body hits the ground (with zero velocity and acceleration as predicted by (6.12)) at $t \simeq 3.3$, whereas at $U = 2$ there is an exponential approach as in (6.14), with $\beta_o \simeq 0.63$, as predicted by (6.16). The curve with $U = 0$ has no physical significance but is included as a limiting case for small U. Figure 4 also shows (dashed) the parabolic free fall in a vacuum, equivalent to (scaled) $m \to \infty$ for all finite U. Fluid-dynamic effects always postpone the impact (sometimes forever!), and also make the impact gentle if it does occur.

7. References

Ames, W. F. 1965. Non-Linear Partial Differential Equations in Engineering, Academic Press.

Bagley, J. A. 1961. The pressure distribution on two-dimensional wings near the ground, A. R. C., R. & M. No. 3238.

Barrows, T. M. 1971. Progress on the ram-wing concept, with emphasis on lateral dynamics, U. S. Dept. of Transportation, Report No. DOT-TSC-FRA-71-7.

Barrows, T. M. & Widnall, S. 1970. The aerodynamics of ram-wing vehicles for application to high-speed ground transportation, A. I. A. A. Paper No. 70-142, 8th Aerospace Sci. Meeting, New York.

Gilbarg, D. 1960. Jets and Cavities, in Handb. der Phys. (ed. Flugge, S.), **9**, 311-438, Springer-Verlag.

Norrbin, N. H. 1974. Bank effects on a ship moving through a short dredged channel, 10th Symp. Naval Hydro., Cambridge, Mass., Office of Naval Research, Washington, D. C.

Pistolesi, E. 1937. Ground effect-theory and practice. N. A. C. A., T. M. no. 828.

Strand, T., Royce, W. W. & Fujita, T. 1962. Cruise performance of channel-flow ground-effect machines, J. Aero. Sci., **29**, 702-711.

Taylor, Sir Geoffrey 1967. Low-Reynolds-Number Flows, 16mm color sound film, Encyclopaedia Britannica Educational Corporation, Chicago, Illinois.

Tuck, E. O. 1975. Matching problems involving flow through small holes, in Advances in Applied Mechanics, (ed. Yih, C. S.), **15**, 89-158, Academic Press.

Tuck, E. O. 1978. Hydrodynamic problems of ships in restricted waters, in Annual Review of Fluid Mechanics, (ed. Van Dyke, M.), **10**, 33-44, Annual Reviews Inc.

Tuck, E. O. & Newman, J. N. 1974. Hydrodynamic interactions between ships, 10th Symp. Naval Hydro., Cambridge, Mass. Office of Naval Research, Washington, D. C.

Widnall, S. E. & Barrows, T. M. 1970. An analytic solution for two- and three-dimensional wings in ground effect, J. Fluid Mech., **41**, 769-792.

Yih, C. S. 1974. Fluid mechanics of colliding plates, Phys. Fluids, **17**, 1936-1940.

Appendix

Edge-Region Matching Problems

The inner one-dimensional flow that is the main topic of the present paper is, in an asymptotic analysis of the complete flow, matched with an outer flow above the body via separate edge-region flows near the bow and stern. For linearized steady flows, the details have been given by Widnall and Barrows (1970), and the present extension to non-linear unsteady flows does not alter the fundamental character of the matching procedure.

In fact, it is clear on stretching that the flow in the edge regions is quasi-steady, in a frame of reference moving with the velocity of the end point. This is because spatial rates of change dominate temporal rates of change by a factor of $0(\epsilon^{-1})$ in the edge regions. For example, the flow at the bow can be written as

$$\phi(x, y, t) = \phi_B(t) - h(t)\, u_B(t)\, \Phi\left(- \frac{x - x_B(t)}{h(t)} , \frac{y}{h(t)} \right) \qquad \text{(A1)}$$

where $\Phi(X, Y)$ is a canonical non-dimensional steady-flow potential, as sketched in Figure 5, and ϕ_B, u_B are the apparent limiting values of ϕ, $\frac{\partial \phi}{\partial x}$, as seen by an inner observer as $x \to x_B + o$. The actual gap size at the bow is written as

$$h(t) = f_L(x_B(t), t) , \qquad \text{(A2)}$$

and this is scaled to a unit value for the non-dimensional potential

$\Phi(X, Y)$. Notice that $\Phi \to X$ as $X \to -\infty$, $o < Y < 1$, so that, on matching with the inner flow as $x \to x_B + o$,

$$\phi(x, t) \to \phi_B + u_B(x - x_B) . \tag{A3}$$

On the other hand, as $\sqrt{X^2 + Y^2} \to \infty$, everywhere other than $X \to -\infty$, $o < Y < 1$,

$$\Phi \to \frac{1}{\pi} \log(\pi e \sqrt{X^2 + Y^2}) \qquad \text{A4)}$$

(the constant "πe" being obtained by conformal mapping solution as in Widnall and Barrows, 1970). Thus, as $\sqrt{x^2 + y^2} \to \infty$,

$$\phi \to (\phi_B - \frac{hu_B}{\pi} \log \frac{\pi e}{h}) - \frac{hu_B}{\pi} \log \sqrt{(x - x_B)^2 + y^2} , \tag{A5}$$

which represents an apparent sink at (x_B, o), as seen by an outer observer, with the correct strength to generate the net flux hu_B entering at the bow. Since the outer flow is everywhere $0(\epsilon)$, the "constant" term in (A5) can only be of order ϵ at most. Since $h = 0(\epsilon)$ already, this means that $\phi_B = 0(\epsilon)$, and hence we can assume as our leading-edge condition on the inner potential $\phi(x, t)$ that $\phi(x_B, t) = 0$, as in the text, This condition was also used by Barrows and Widnall (1970).

Figure 5 is drawn for leading edges without $0(1)$ curvature. However, curved leading edges do not alter the qualitative features

of the above argument, only the value of the constant replacing "πe" in (A4). In particular, the conclusion $\phi_B = 0(\epsilon)$ still applies for curved leading edges, so long as the curvature remains 0(1) only within $0(\epsilon)$ of the edge.

The flow near the stern is <u>not</u> like that in Figure 5, since we assume that the stern flow separates smoothly into a wake, rather than negotiating the sharp edge to become source-like at infinity. In fact, if the trailing under-surface does not possess 0(1) edge curvature, the stern flow is extremely simple, consisting of nothing more than a uniform stream emerging from beneath a parallel plane, with a constant wake height equal to the trailing-edge height $f_L(x_S, t)$. Thus, if $x - x_S = 0(\epsilon)$, the stern edge flow is given by

$$\phi(x, y, t) = \phi_S(t) + u_S(t)(x - x_S(t)) + 0(\epsilon^2) \tag{A6}$$

which matches the inner flow, with ϕ_S, u_S as the apparent trailing-edge values of ϕ, $\frac{\partial \phi}{\partial x}$ respectively. The wake condition (2.10) for $x > x_S$ reduces to (3.12) in this case, since $p(x, f_W + o, t)$ and $\phi_y = o$, and is satisfied with $0(\epsilon)$ error, providing

$$\dot{\phi}_S - u_S \dot{x}_S + \frac{1}{2} u_S^2 = 0 , \tag{A7}$$

This is equivalent to the Kutta condition for the inner flow, i.e. to (3.12) applied at $x = x_S(t) - 0$, since

$$\dot{\phi}_S = \frac{d}{dt}\,\phi(x_S(t), t) = \dot{\phi}(x_S(t), t) + u_S \dot{x}_S \ . \qquad \text{(A8)}$$

Somewhat more interesting is the case where the lower-surface geometry near the stern is curved to $0(1)$ slopes, as sketched in Figure 6. The important gap size is then the apparent trailing edge height $h(t)$ seen by an inner observer, which matches with an apparent gap width as $x \to -\infty$, seen by an observer in the stern-edge region, i.e.

$$h(t) = \lim_{\substack{x \to x_S - 0 \\ (x = 0(1))}} \left[f_L(x,t) \right] = \lim_{\substack{x \to -\infty \\ (x - x_S = 0(\epsilon))}} \left[f_L(x,t) \right] \ . \qquad \text{(A9)}$$

Now we can write the trailing-edge solution as

$$\phi(x,y,t) = \phi_S(t) + h(t)U(t)\,\Phi\left(\frac{x - x_S(t)}{h(t)}, \frac{y}{h(t)}\right) + (x - x_S(t))\dot{x}_S(t), \qquad \text{(A10)}$$

where $\Phi(X, Y)$ is the canonical steady-flow potential sketched in Figure 6, and ϕ_S, U are to be determined. Note that Φ possesses a unit velocity magnitude as $X \to +\infty$. We assume that the wake for $\Phi(X, Y)$ is bounded by a free streamline $Y = F(X)$ which is of width σ at $X = +\infty$, relative to the unit-width channel at $X = -\infty$, and on which the flow generated by Φ has a unit magnitude. The last term in (A10) is needed to bring the moving body surface to rest.

The problem for $\Phi(X, Y)$ is a classical free-streamline problem (c.f. Gilbarg, 1960), and hodograph techniques may be used to solve

for Φ, for any given stern geometry. In particular, we then obtain the coefficient σ, which is commonly called the "coefficient of contraction" for the given geometry. Actual contraction, as in a nozzle, applies when $\sigma < 1$; Figure 6 illustrates an "expansion" case with $\sigma > 1$. If a rudder is present, one would expect $\sigma < 1$ when the rudder is to port and $\sigma > 1$ when it is to starboard. The constant C shown in Figure 6 can also be computed, but plays no essential role, to leading order in ϵ.

Verification of the validity of (A10) demands only that ϕ matches with the inner flow as $\frac{x - x_S}{h} \to -\infty$, and satisfies the unsteady wake-boundary condition (2.10). Clearly, by its construction, $\phi_S(t)$ is identifiable with $\phi(x_S(t), t)$, and since $\Phi \to \sigma X$ as $X \to -\infty$,

$$\phi'(x_S(t), t) = \sigma U + \dot{x}_S \quad . \tag{A11}$$

It is sufficient to check the pressure condition as $X \to +\infty$, giving (retaining only $0(1)$ terms, and hence neglecting the constant C in $\Phi \to X + C$ as $X \to +\infty$, as well as $0(\epsilon)$ terms such as $\dot{U}(x - x_S)$, etc.),

$$(\dot{\phi}_S - U\dot{x}_S - \dot{x}_S^2) + \frac{1}{2}(U + \dot{x}_S)^2 = 0$$

or

$$\dot{\phi}_S + \frac{1}{2}U^2 = \frac{1}{2}\dot{x}_S^2 \quad . \tag{A12}$$

Substituting for U from (A11) and $\dot{\phi}_S$ from (A8), we have (with $\phi = \phi(x_S, t)$, $u_S = \phi'(x_S, t))$,

$$\dot{\phi} + u_S \dot{x}_S + \frac{1}{2\sigma^2} (u_S - \dot{x}_S)^2 = \frac{1}{2} \dot{x}_S^2 \tag{A13}$$

which is equivalent to (3. 14), after some manipulation.

FIGURE CAPTIONS

Figure 1. Sketch of flow and geometry.

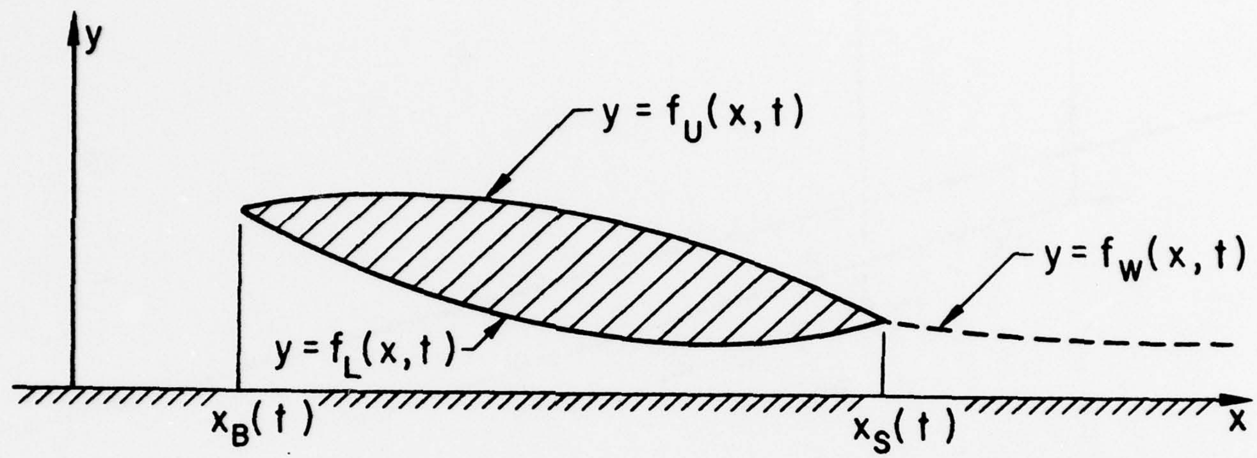
Figure 2. Transient wake velocity and thickness at $\mu = 3$, for an airfoil started impulsively at $t = o$ with constant speed U, plotted against x for various times t.

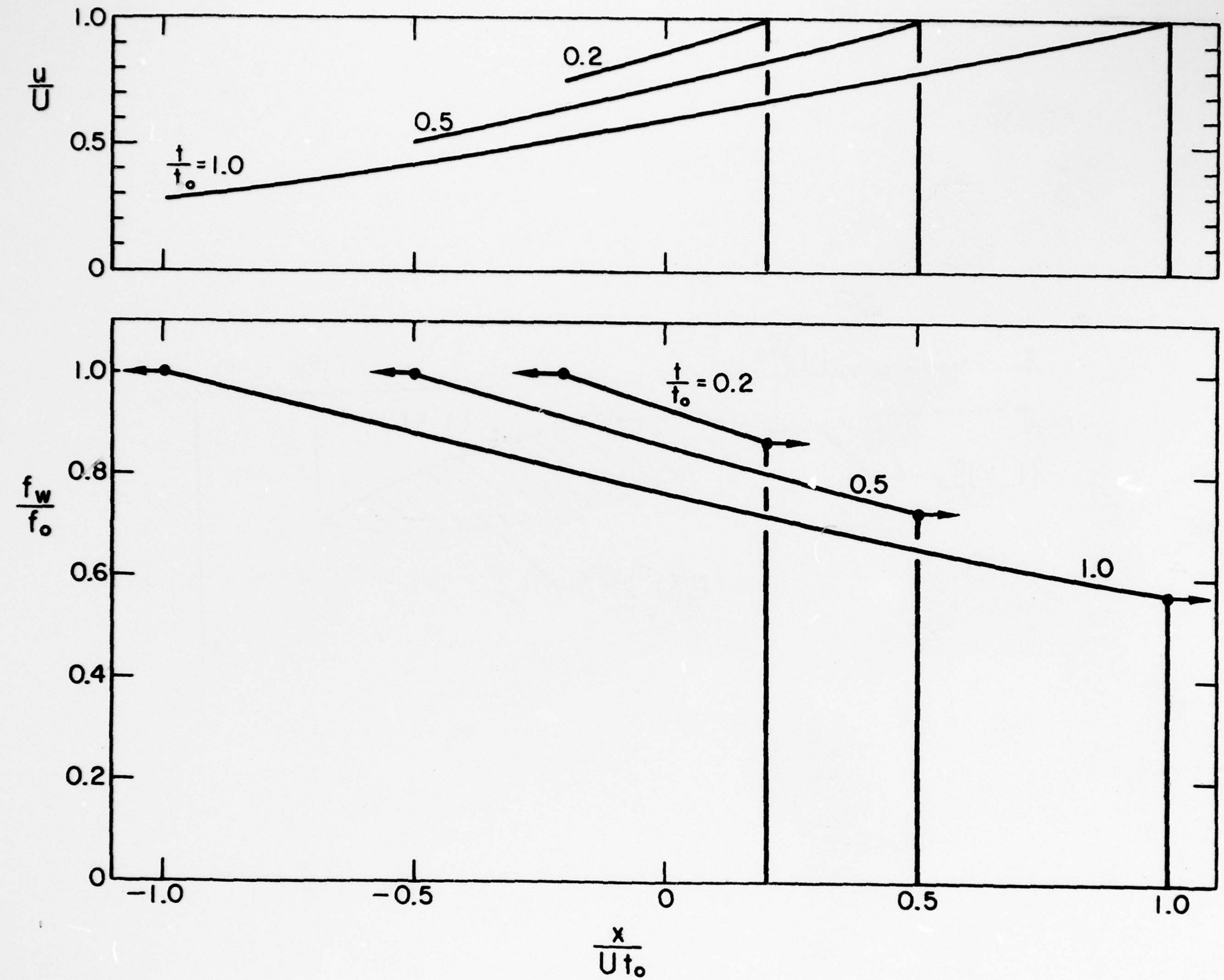
Figure 3. Transient wake as in Figure 2, at $\mu = -3$. Note "shock" ($f_W \rightarrow \infty$) for $t/t_o = 2, 4$.

Figure 4. Scaled height-versus-time trajectories for fall of a plate of (scaled) mass $m = 1$, for various (scaled) horizontal speeds U. Arrows indicate times of impact for $U = 0, 1$; there is no impact if $U > 1.64$.

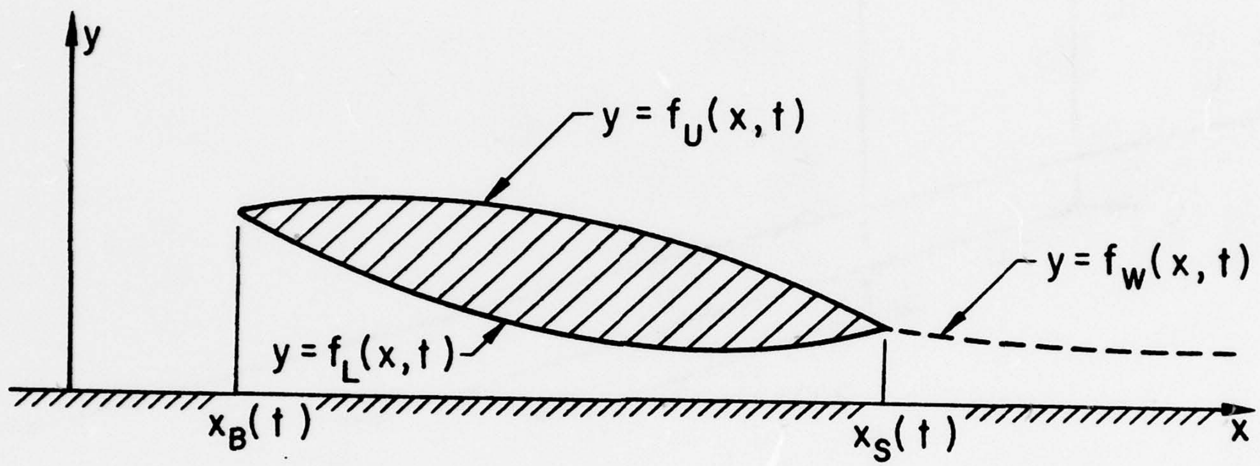
Figure 5. Canonical unit-velocity flow from a unit-width straight channel, emerging into a half-plane, as a source of strength 2 at infinity.

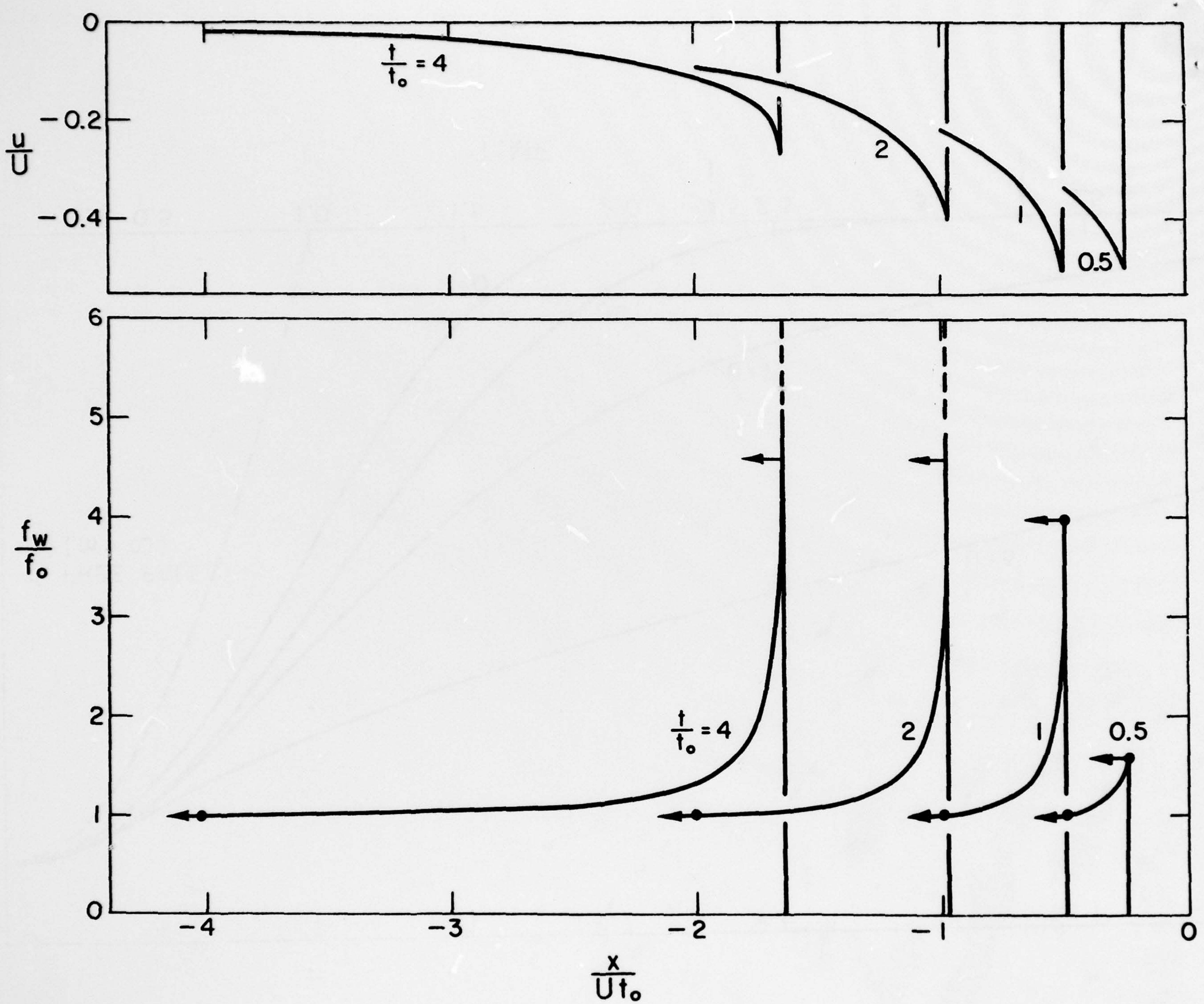
Figure 6. Canonical free-streamline flow near shaped stern, or stern with rudder. Channel flow has velocity σ and width 1, whereas wake at infinity has velocity 1 and width σ, where σ is the contraction coefficient of the effective nozzle so formed.


y
y = f_U(x, t)
y = f_W(x, t)
y = f_L(x, t)
x_B(t)
x_S(t)
x

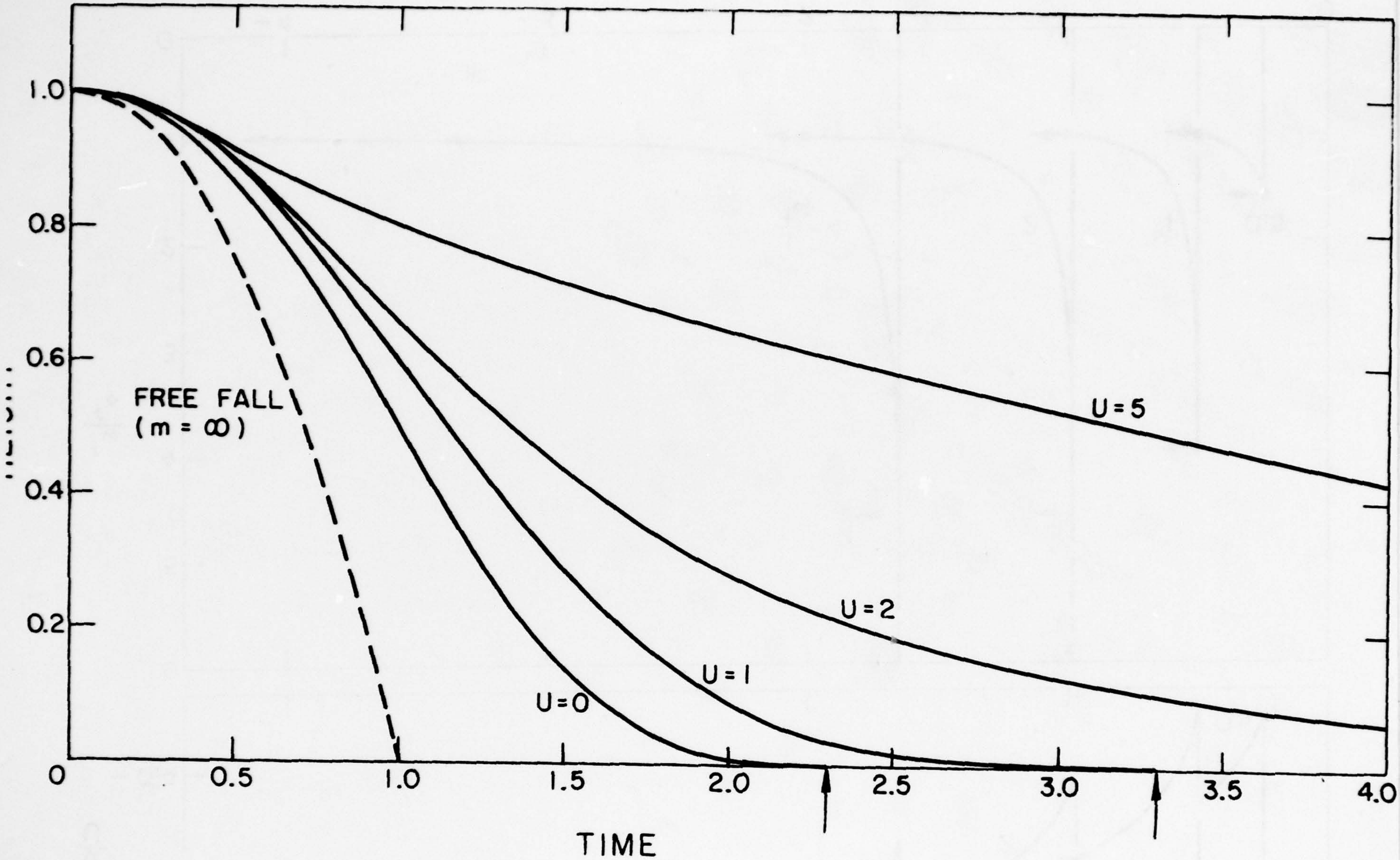


u/U
1.0
0.5
0
0.2
0.5
t/to = 1.0
fw/fo
1.0
0.8
0.6
0.4
0.2
0
t/to = 0.2
0.5
1.0
−1.0
−0.5
0
0.5
1.0
x/Uto

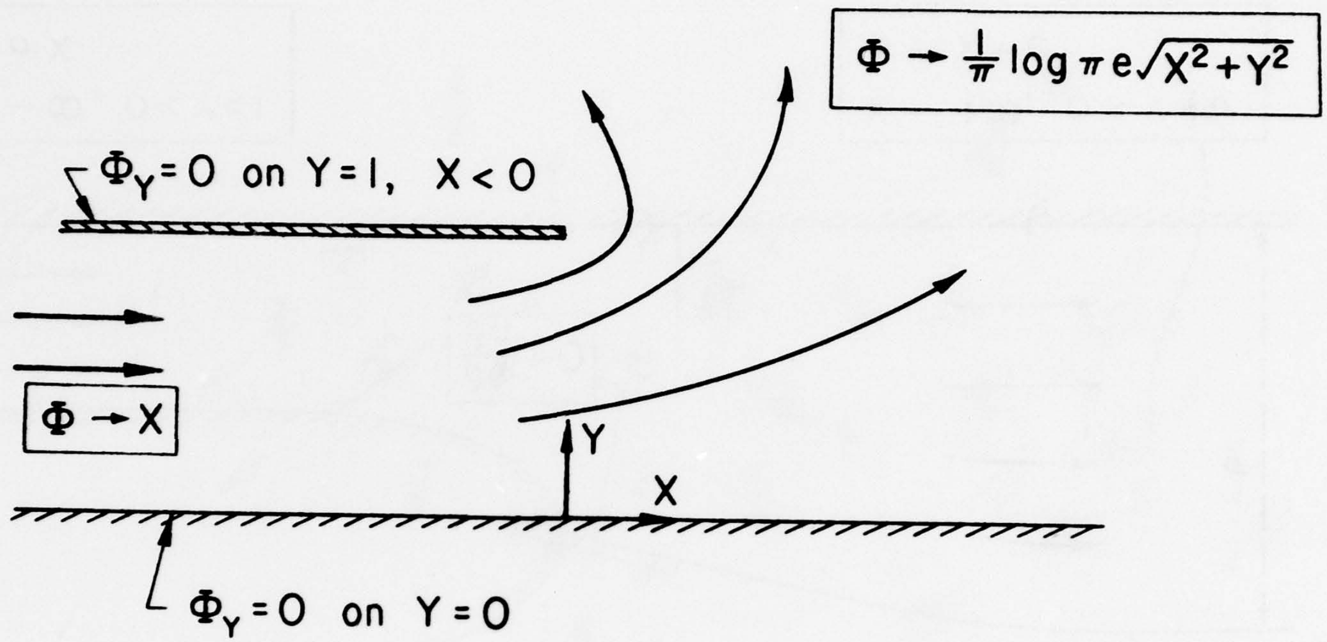

y
y = f_U(x, t)
y = f_W(x, t)
y = f_L(x, t)
x_B(t)
x_S(t)
x



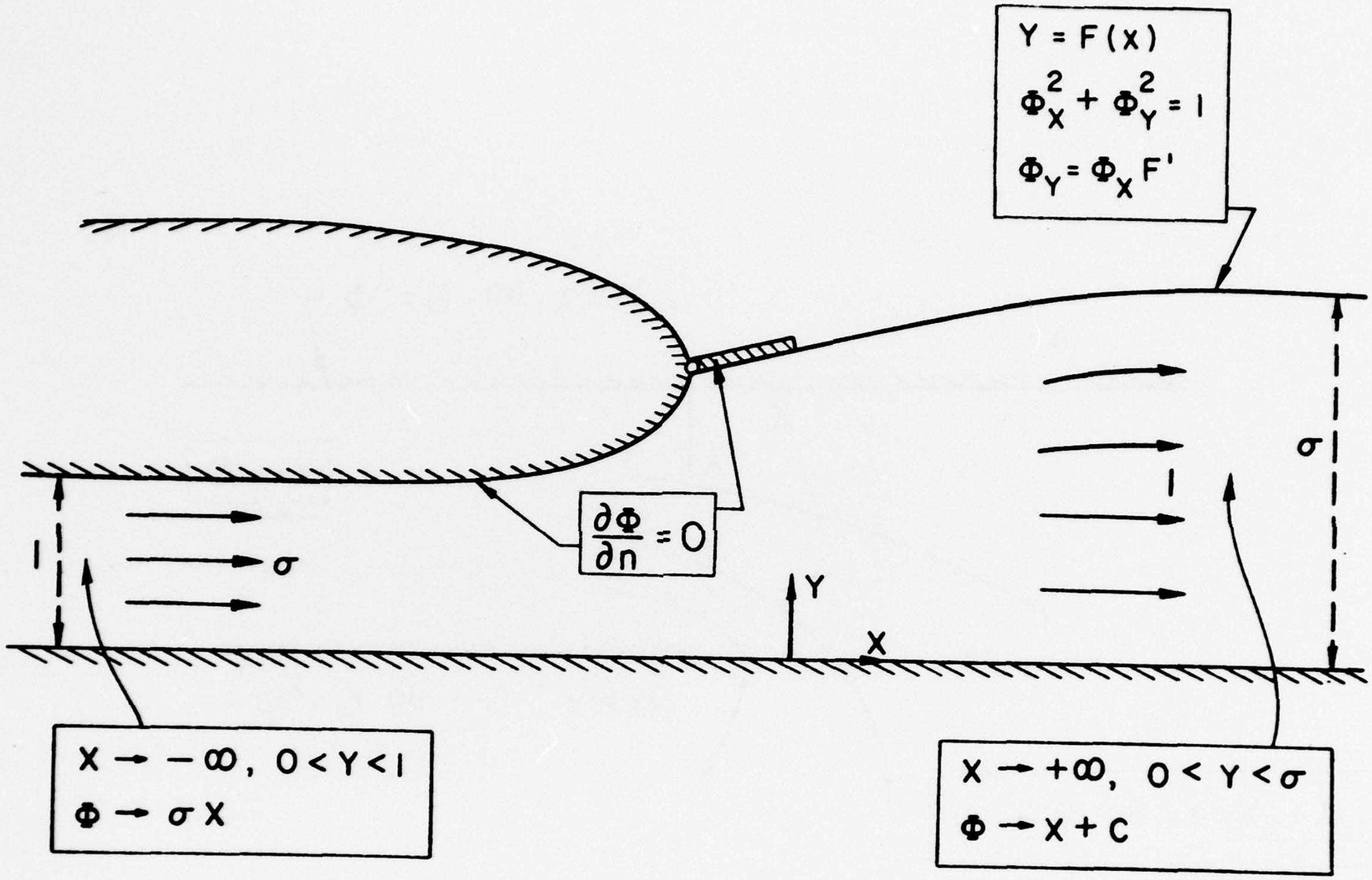
u/U
0
-0.2
-0.4
t/to = 4
2
1
0.5
fw/fo
6
5
4
3
2
1
0
t/to = 4
2
1
0.5
-4
-3
-2
-1
0
x/Uto



1.0
0.8
0.6
0.4
0.2
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
TIME
FREE FALL
(m = ∞)
U=0
U=1
U=2
U=5



$\Phi \to \frac{1}{\pi} \log \pi e \sqrt{X^2+Y^2}$
$\Phi_Y = 0$ on $Y = 1,\ X < 0$
$\Phi \to X$
Y
X
$\Phi_Y = 0$ on $Y = 0$



Y = F(X)
Φ_X^2 + Φ_Y^2 = 1
Φ_Y = Φ_X F'
σ
1
∂Φ/∂n = 0
1
σ
Y
X
X → −∞, 0 < Y < 1
Φ → σX
X → +∞, 0 < Y < σ
Φ → X + C

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: E-97B-54	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): UNSTEADY SMALL-GAP GROUND EFFECTS	5. TYPE OF REPORT & PERIOD COVERED: Technical report
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): E. O. Tuck	8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0157
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Division of Engineering and Applied Science, California Institute of Technology, Pasadena, California 91125	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR 062-230
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Department of the Navy, Arlington, Virginia 22217	12. REPORT DATE: March 1979
	13. NUMBER OF PAGES: 44
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office): ONR Branch Office, 1030 East Green Street, Pasadena, California 91101	15. SECURITY CLASS. (of this report): unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this publication in unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Unsteady lifting-surface
Ground effects

(cont fr p3)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Flow induced by a body moving near a plane wall is analysed on the assumption that the normal distance from the wall of every point of the body is small compared to the body length. The flow is irrotational except for the vortex sheet representing the wake. The gap-flow problem in the case of unsteady motion is reduced to a nonlinear first-order ordinary differential equation in the time variable. Problems solved include airfoil starting flows and their transient wakes, and flat plates falling toward the ground.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)